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1997 J. Phys. A: Math. Gen. 30 5697

(http://iopscience.iop.org/0305-4470/30/16/013)

J. Phys. A: Math. Gen. **30** (1997) 5697–5707. Printed in the UK PII: S0305-4470(97)79878-0

Fluctuation-controlled transient below the on–off intermittency transition

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Received 26 November 1996, in final form 25 March 1997

Abstract. On–off intermittency is an ubiliquitous phenomenon observed beyond the instability of a particular chaotic motion. The system below the instability reveals a typical transient chaos. It is found that near the instability point the inverse characteristic first passage time α satisfies the scaling law $\alpha = (\lambda_\perp^2/4D_\perp)g(|\lambda_\perp|/|\lambda_\perp^*|)$ with the transverse Lyapunov exponent (TLE) $\lambda_\perp(< 0)$, $\lambda_\perp^*$ and $D_\perp$ being respectively an initial condition-dependent characteristic TLE and the fluctuation intensity of local transverse expansion rate. Numerical simulation for several systems suggests that the scaling function $g(z)$ is a universal function.

1. Introduction

Intermittency is a typical, highly nonlinear phenomenon and is observed in any field of nonlinear science. In low-dimensional dynamical systems, three types of intermittent temporal evolution of dynamical variable were established by Pomeau and Manneville in connection with the instability of periodic trajectories [1]. Recently a different kind of intermittency, due to the instability of a particular chaotic motion, has attracted much attention [2–15]. In contrast to that, the PM intermittency is observed after the instability of periodic orbits, this intermittency is observed when a particular chaotic state becomes unstable. This intermittency is called the *on–off intermittency* after the specific temporal evolution of dynamical variables, and is observed not only in numerical simulations [2–11] but also in laboratory experiments, expecially in electronic circuits [12–15].

Although many studies have recently been carried out to clarify the statistical characteristics of the on–off intermittency, less contribution is performed on the other side of on–off intermittency. Below the on–off intermittency transition called the blowout bifurcation in [7], the system exhibits a typical transient chaos. The fundamental aim of this paper is to report a new statistical law characterizing the transient below the intermittency transition.

The paper is organized as follows. In section 2, the onset mechanism and the statistical characteristics of on–off intermittency are briefly reviewed. In section 3, introducing the first passage time which characterizes the transient, we propose a new scaling law for the characteristic first passage time slightly below the transition. Carrying out numerical simulations of several models, we will show that the scaling property is an ubiquitous phenomenon associated with the precursor of on–off intermittency. Concluding remarks are given in section 4.

0305-4470/97/165697+11$19.50

2. On–off intermittency—Onset mechanism and statistics

Let us consider the combined system of dynamical variables $\boldsymbol{X}$ and $\boldsymbol{u}$,

$$\dot{\boldsymbol{X}}(t) = \boldsymbol{F}(\boldsymbol{X}(t)) + \boldsymbol{f}(\boldsymbol{X}(t), \boldsymbol{u}(t)) \tag{2.1a}$$

$$\dot{\boldsymbol{u}}(t) = \boldsymbol{g}(\boldsymbol{X}(t), \boldsymbol{u}(t)) \tag{2.1b}$$

where $\boldsymbol{F}(\boldsymbol{X})$ is a nonlinear function of $\boldsymbol{X}$, and $\boldsymbol{f}$ and $\boldsymbol{g}$ are analytical functions of $\boldsymbol{X}$ and $\boldsymbol{u}$, satisfying

$$\boldsymbol{f}(\boldsymbol{X}, \boldsymbol{0}) = \boldsymbol{g}(\boldsymbol{X}, \boldsymbol{0}) = \boldsymbol{0} \tag{2.2}$$

for any choice of $\boldsymbol{X}$ and parameter values. The equations of motion (2.1) yield the particular motion obeying

$$\dot{\boldsymbol{X}}^0(t) = \boldsymbol{F}(\boldsymbol{X}^0(t)) \qquad \boldsymbol{u}^0(t) = \boldsymbol{0}. \tag{2.3}$$

For the coupled oscillator system (equation (3.3)), $\boldsymbol{X}$ stands for average variables of two oscillators and $\boldsymbol{u}$ is the difference of two variables. In the Ott–Sommerer model (equation (3.5)), $\boldsymbol{X}$ is the variables $(x, \dot{x})$ and $\boldsymbol{u}$ is $(y, \dot{y})$. The region in the phase space, where the phase point is given by (2.3) is hereafter called the *invariant manifold*.

The stability of the particular motion (2.3) is examined by observing how the distance from the invariant manifold changes in time. Let us define two variables $\boldsymbol{u}_\parallel$ and $\boldsymbol{u}_\perp$ by

$$\dot{\boldsymbol{u}}_\mu(t) = \hat{G}_\mu(t)\boldsymbol{u}_\mu(t) \qquad (\mu = \parallel, \perp) \tag{2.4}$$

with the perturbation matrices

$$\hat{G}_\parallel(t) = \frac{\partial \boldsymbol{F}(\boldsymbol{X})}{\partial \boldsymbol{X}}\Big|_{\boldsymbol{X}=\boldsymbol{X}^0(t)} \qquad \hat{G}_\perp(t) = \frac{\partial \boldsymbol{g}(\boldsymbol{X}^0(t), \boldsymbol{u})}{\partial \boldsymbol{u}}\Big|_{\boldsymbol{u}=\boldsymbol{0}}. \tag{2.5}$$

By putting $l_\mu(t) = |\boldsymbol{u}_\mu(t)|$, (2.4) yields

$$\dot{l}_\mu(t) = \Lambda_\mu^0(t) l_\mu(t). \tag{2.6}$$

$l_\parallel(t)$ evaluates the nearby distance on the invariant manifold. We introduce two exponents $\lambda_\parallel$ and $\lambda_\perp$ by

$$\lambda_\mu = \lim_{t\to\infty} \frac{1}{t} \log \frac{l_\mu(t)}{l_\mu(0)} = \lim_{t\to\infty} \frac{1}{t} \int_0^t \Lambda_\mu^0(s)\, \mathrm{d}s. \tag{2.7}$$

The exponent $\lambda_\parallel$ is relevant to the trajectory stability on the invariant manifold, and is identical to the ordinary largest Lyapunov exponent of the dynamics $\dot{\boldsymbol{X}} = \boldsymbol{F}(\boldsymbol{X})$. On the other hand, since $l_\perp(t)$ evaluates the distance of the phase point from the invariant manifold, $\lambda_\perp$ measures the stability of the particular motion obeying (2.3). $\lambda_\perp$ is called the *stability parameter* or the *transeverse Lyapunov exponent* (TLE). Let us put

$$\Lambda_\perp^0(t) = \lambda_\perp + f(t) \tag{2.8}$$

where $\lambda_\perp$ is the average, being identical to that in (2.7), and $f(t)$ is the fluctuation of $\Lambda_\mu^0(t)$. The particular motion on the invariant manifold being set to be chaotic ($\lambda_\parallel > 0$), the fluctuation $f(t)$ is assumed to have a mixing property. The quantity

$$D_\perp \equiv \int_0^\infty \langle f(t) f(0)\rangle\, \mathrm{d}t \qquad (> 0) \tag{2.9}$$

measures the intensity of the fluctuation of the *local transverse expansion rate* (LTER) $\Lambda_\mu^0(t)$. Although $\lambda_\perp$ changes its sign at the instability point, it is expected that the fluctuation statistics of $\Lambda_\perp^0(t)$ is not sensitive to the distance from the intermittency transition. So, we assume that $D_\perp$ is constant near the instability point†. For $\lambda_\perp < 0$ (> 0), the particular

† The quantity $D_\parallel \equiv \int_0^\infty \langle (\Lambda_\parallel^0(t) - \lambda_\parallel)(\Lambda_\parallel^0(0) - \lambda_\parallel)\rangle\, \mathrm{d}t$ measures the intensity of the fluctuation of local expansion rate on the invariant manifold, i.e. on the strange attractor for the dynamics $\dot{\boldsymbol{X}}(t) = \boldsymbol{F}(\boldsymbol{X}(t))$.

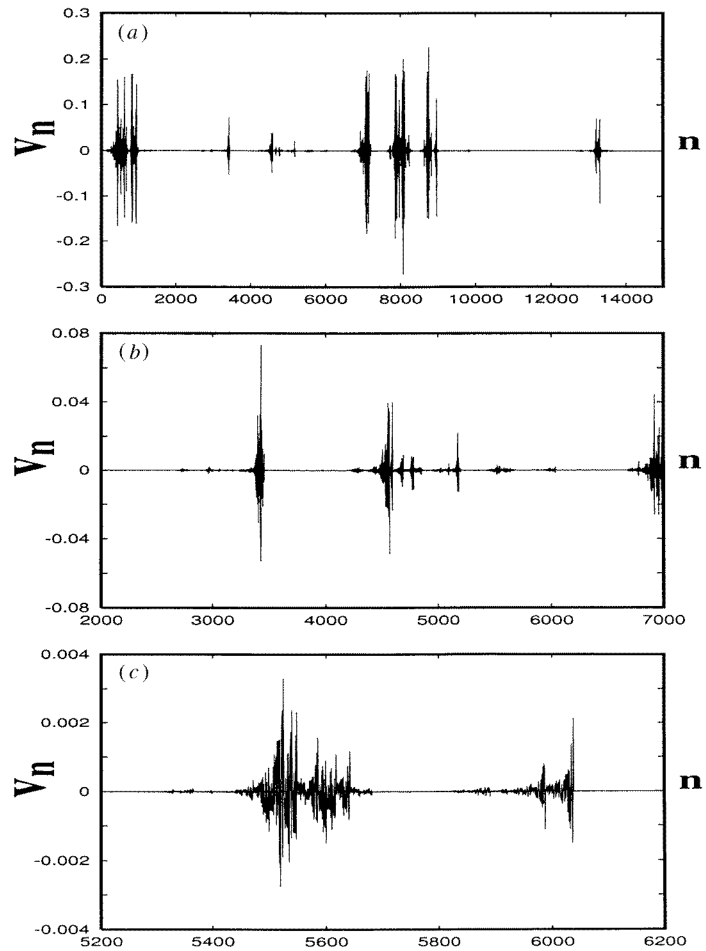

Figure 1. (a) The temporal evolution of on–off intermittency variable $v_n \equiv X_n^{(1)} - X_n^{(2)}$ in the coupled logistic map system (3.3) with $a = 3.8$ and $K = 0.4271$, ($\lambda_\perp = 0.0050$) after a sufficiently long transient. (b) The blow-up of (a) and (c) is the blow up of (b). These manifest a self-similar evolution of on–off intermittency variable.

motion is stable (unstable). For $\lambda_\perp < 0$, the phase point eventually approaches the invariant manifold.

It is well known that after the instability of the particular chaotic motion as the external control parameter is changed, the system usually exhibits the on–off intermittency (see figure 1). Phenomenologically necessary conditions on the onset of on–off intermittency are summarized as follows. The first is that TLE changes its sign from nagative to positive [2–4]. The second is that LTER, whose statistical average is TLE, exhibits a fluctuation in the sense that $D_\perp$ takes a positive, finite value. In [2], we studied the breakdown of the synchronization in a coupled map system whose elements consisted of fully developed logistic parabola. Since the fully developed logistic parabola is transformed into the tent map, the quantity $D_\perp$ rigorously vanishes. We did not observe the on–off intermittent characteristics. Thirdly, the transition should be continuous in the sense that all statistical quantities continuously change at the transition point. It is remarked that the breakdown of the particular solution as the control parameter is changed does not necessarily lead to the on–off intermittency. Previously we studied the breakdown of the synchronization in the coupled chaos system whose elements consist of the Lorenz model [16]. For certain parameter values, after the breakdown of the synchronization we observed that the system falls into a fixed point. The transition is discontinuous and a hysteresis is observed.

In order to study the statistical law of the on–off intermittency we phenomenologically introduce the stochastic model for the distance $l(t)$ of the phase point from the invariant manifold. Assume that $l(t)$ obeys [17, 18]

$$\dot{l}(t) = \Lambda_\perp(t) l(t) \tag{2.10}$$

$$\Lambda_\perp(t) = \lambda_\perp + f(t) - \beta l(t)^m. \tag{2.11}$$

Here $\lambda_\perp$ and $f(t)$ are same as in (2.8), β is a positive constant and m is a positive integer. Furthermore, in order to make the problem tractable, $f(t)$ is assumed to be a Gaussian-white noise with zero mean and the intensity $D_\perp$ (equation (2.9)). This makes the problem complete. Statistical laws known until now for the on–off intermittency can be derived from the multiplicative noise model [17, 18]. Namely, the distribution for $l(t)$, the distance from the invariant manifold, takes the power law

$$P(l) \sim l^{-1+\eta} \qquad \eta = \frac{\lambda_\perp}{D_\perp} \tag{2.12}$$

for small l. The spectral intensity for the time series $\{l(t)\}$ obeys the power law

$$I(\omega) \sim \omega^{-\frac{1}{2}} \tag{2.13}$$

for small ω [17, 18, 7, 8]. Furthermore, the laminar length distribution $Q(\tau)$ [6] obeys

$$Q(\tau) \sim \tau^{-\frac{3}{2}}. \tag{2.14}$$

Very recently, Hata and Miyazaki [19] constructed a solvable dynamical model of on–off intermittency, and rigorously found the power law (2.12) and the asymptotics (2.14). It seems that (2.14) is a universal law. We recently found that in a four-dimensional coupled map system the exponent slightly changes as a function of the control parameter [10], and slightly increases as the intermittency develops. This may suggest the variety of universality classes of on–off intermittency.

3. Transient below the intermittency transition

On the other side of the on–off intermittency, i.e. below the transition ($\lambda_\perp < 0$) the system eventually approaches the particular chaos restricted on the invariant manifold with $l = 0$. In this process, the system shows a typical transient chaos [20]. Let us introduce the *first passage time* (FPT) t_{FPT} when $l(t_{\mathrm{FPT}}) = l_c$ first holds for a given l_c by starting with $l(0) \equiv l_0 (> l_c)$. FPT is a function of the initial state. Various initial values produce a distribution of FPT. Let us define the probability distribution for t_{FPT} as

$$W(\tau) \equiv \langle \delta(t_{\mathrm{FPT}} - \tau) \rangle. \tag{3.1}$$

Here, in dynamical systems, $\langle \cdots \rangle$ is the average over the initial ensemble uniformly distributed near the on–off intermittency attractor region for $\lambda_\perp < 0$ with the constraint that l_0 and l_c are given. In the multiplicative noise model, the average should be taken over a whole realization of random noise for given l_0 and l_c. Figure 2 displays the distribution of initial-state points giving different lengths of FPT. One finds a complicated *riddled structure* in the state space [7, 21]. Namely, an infinitesimal change of the initial condition produces a

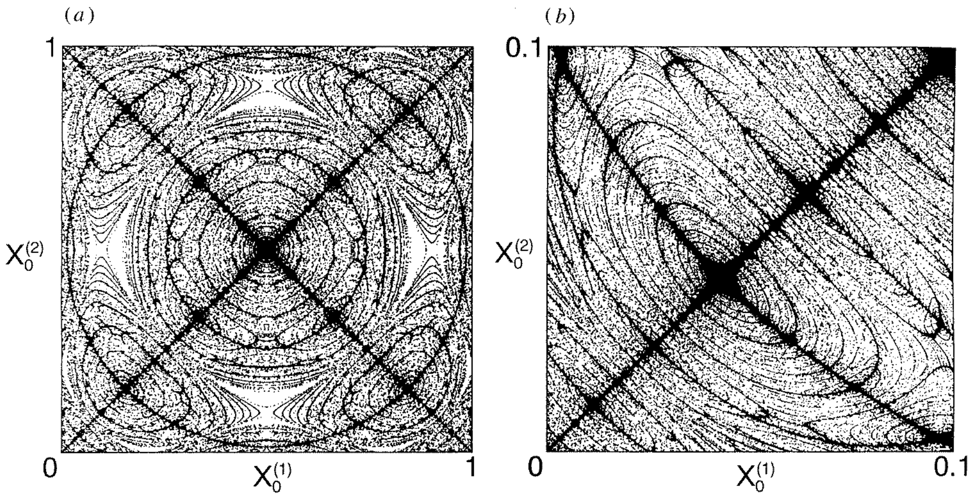

Figure 2. (a) FPT distribution as a function of initial point $(X_0^{(1)}, X_0^{(2)})$ for the coupled logistic map system (3.3) slightly below the on–off intermittency transition, ($a = 3.8$, $K = 0.433 > K_c(= 0.4321)$, $l_c = 10^{-3}$). Black points correspond to initial points giving FPT longer than 20. (b) The blow-up of (a). In each figure, $10^3 \times 10^3$ initial points were prepared. A slight change of the initial condition causes a huge change of first passage time. This is called the riddled structure of geometrical structure in the state space.

huge difference of FPT, (*sensitive dependence of FPT and the transverse Lyapunov exponent over a time interval on initial condition*).

Figure 3(a) displays how the distribution changes by changing l_c and the control parameter by keeping l_0 fixed. One finds that the distribution has a single peak and its position and width depend on both the initial distance l_0 and the final distance l_c as well as the distance of the control parameter from the instability point. The transient process is characterized by the FPT distribution. The distribution for relatively short times crucially depends on the system under consideration, and is not universal. On the other hand, it takes the exponential form

$$W(\tau) \propto e^{-\alpha\tau} \tag{3.2}$$

for large τ, where α^{-1} is the characteristic FPT. α characterizes the transient process on the other side of the on–off intermittency and reflects the chaotic dynamics of the particular motion, and generally depends on l_0, l_c and $\lambda_\perp$ (< 0) which evaluates the distance from the transition point. Figures 3(b) and 3(c) roughly show how α changes depending on l_c and $\lambda_\perp$. α is approximately independent of l_c if $|\lambda_\perp|$ is appropriately large, while α crucially depends on l_c provided that $|\lambda_\perp|$ is sufficiently small.

Hereafter we will discuss how α depends on l_c and $\lambda_\perp$ near the instability point by carrying out numerical simulations by keeping l_0 a typical value for simplicity. Numerical models are given below. These models can be transformed into the standard form (2.1) with a linear transformation, and show on–off intermittency.

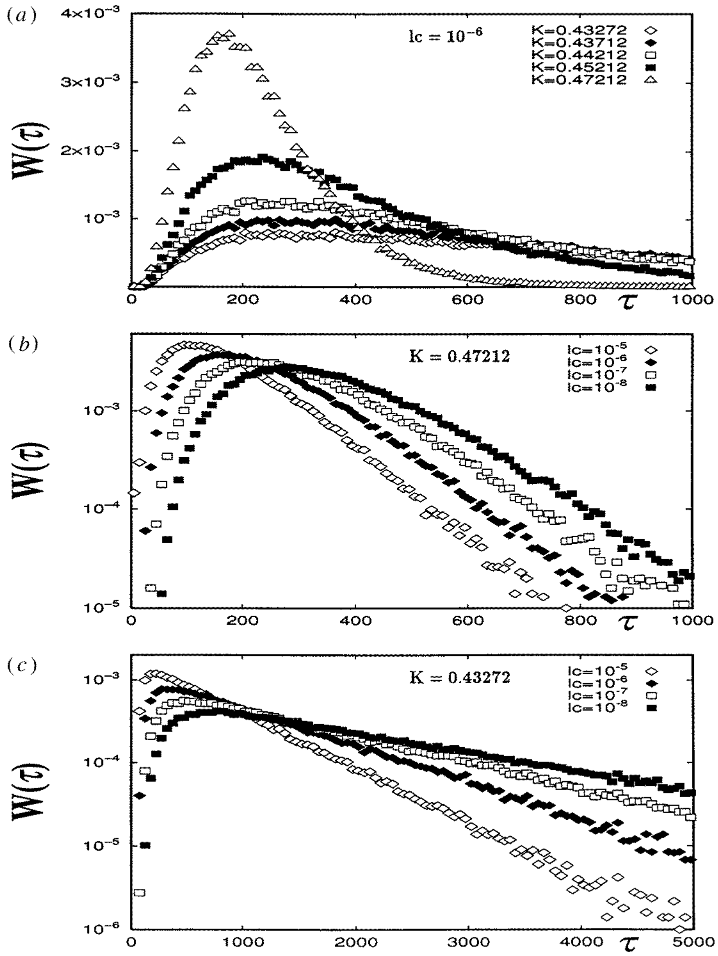

Figure 3. FPT distributions for different values of l_c and K in the coupled logistic map system (3.3) with $a = 3.8$. We put $l_0 = 10^{-2}$. The inverse characteristic FPT, α, is independent of l_c sufficiently below the transition point ($K > K_c$)((*b*)), and depends on l_c near the transition point($K \approx K_c$)((*c*)).

3.1. Model A. Coupled map system

Consider the coupled map system [22, 23]

$$X_{n+1}^{(j)} = F(X_n^{(j)}) + \xi \sum_{l=1}^{2} \{F(X_n^{(l)}) - F(X_n^{(j)})\} \qquad (j = 1, 2) \tag{3.3}$$

with $\xi = (1 - \mathrm{e}^{-K})/2$, $(K > 0)$†. The motion on the invariant manifold is the synchronized oscillation $X_{n+1}^0 = F(X_n^0)$. In this paper we use the logistic parabola $F(X) = aX(1 - X)$ with $a = 3.8$, where $\lambda_\parallel = 0.4321$. TLE is given by

$$\lambda_\perp = \lambda_\parallel - K \tag{3.4}$$

† For coupled map systems, the standard form (2.1) is written as $\boldsymbol{X}_{n+1} = \boldsymbol{F}(\boldsymbol{X}_n) + \boldsymbol{f}(\boldsymbol{X}_n, \boldsymbol{u}_n)$, $\boldsymbol{u}_{n+1} = \boldsymbol{g}(\boldsymbol{X}_n, \boldsymbol{u}_n)$ with $\boldsymbol{f}(\boldsymbol{X}, \boldsymbol{0}) = \boldsymbol{g}(\boldsymbol{X}, \boldsymbol{0}) = \boldsymbol{0}$. The particular solution always obeys $\boldsymbol{X}_{n+1}^0 = \boldsymbol{F}(\boldsymbol{X}_n^0)$, $\boldsymbol{u}_n^0 = \boldsymbol{0}$. In model (3.3), putting $X_{n+1} = (X_n^{(1)} + X_n^{(2)})/2$, $u_n = (X_n^{(1)} - X_n^{(2)})/2$, one obtains the standard form.

where $\lambda_{\parallel}$ is the Lyapunov exponent of the one-dimensional map $X_{n+1} = F(X_n)$. If the system parameter is chosen such that $\lambda_{\parallel}$ is positive, the system undergoes an on–off intermittency transition at $K = \lambda_{\parallel} (\equiv K_c)$ as the coupling constant K is decreased.

3.2. Model B. Ott–Sommerer model

This is the two-dimensional particle motion subject to the external periodic excitation in the x-direction obeying [8, 9]

$$\ddot{\boldsymbol{r}}(t) = -\nu \dot{\boldsymbol{r}}(t) - \nabla U(\boldsymbol{r}) + f \sin(\Omega t) \boldsymbol{e}_x \tag{3.5}$$

$$U(\boldsymbol{r}) = (1 - x^2)^2 + y^2(x - p) + k y^4 \tag{3.6}$$

where $\boldsymbol{r} = (x, y)$ is the particle position, ν is the damping constant, k is a non-negative constant, f and Ω are respectively the amplitude and the angular frequency of the external force. The system has a particular motion on the x-axis. When the particular motion is chaotic, after the instability of the particular motion at $p = p_c$, the particle starts to intermittently move in the y-direction. This shows a typical on–off intermittency. For parameter values $\nu = 0.05$, $f = 2.3$, $\Omega = 3.5$, $k = 0$, we obtain $p_c = -1.7887$.

3.3. Model C. Multiplicative noise model

This is given by (2.10) with $m = 2$ for $\lambda_{\perp} < 0$.

As discussed in the previous section, the statistical characteristics of the on–off intermittency is well described by the multiplicative noise model (2.10) with (2.11). After a sufficiently long time for any initial condition, $l(t)$ becomes small. So we first ignore the nonlinear term with respect to the deviation from the particular motion. If $l(t)$ is small enough, (2.11) is replaced by (2.8). As shown in the theory of first passage time [24, 25], the normalized distribution for the Gaussian-white random noise $f(t)$ rigorously takes the form

$$W_0(\tau) = \frac{1}{\sqrt{4\pi D_{\perp}}} \frac{\log(l_0/l_c)}{\tau^{3/2}} \exp\left[-\frac{(\lambda_{\perp}\tau + \log(l_0/l_c))^2}{4 D_{\perp} \tau} \right]. \tag{3.7}$$

This yields

$$\alpha = \frac{\lambda_{\perp}^2}{4 D_{\perp}}. \tag{3.8}$$

Furthermore, moments are given by

$$\langle \tau^{\nu} \rangle_0 \equiv \int_0^{\infty} \tau^{\nu} W_0(\tau)\, \mathrm{d}\tau = \left(\frac{\log(l_0/l_c)}{|\lambda_{\perp}|} \right)^{\nu} \left(\frac{2|\lambda_{\perp}|}{\pi |\lambda_{\perp}^0|} \right)^{\frac{1}{2}} \mathrm{e}^{|\lambda_{\perp}|/|\lambda_{\perp}^0|} K_{\nu - \frac{1}{2}} \left(\frac{|\lambda_{\perp}|}{|\lambda_{\perp}^0|} \right) \tag{3.9}$$

where

$$|\lambda_{\perp}^0| \equiv \frac{2 D_{\perp}}{\log(l_0/l_c)} \tag{3.10}$$

is the characteristic value of TLE, and $K_{\nu}(z)$ is the modified Bessel function. Particularly, $\langle \tau \rangle_0 = \log(l_0/l_c)/|\lambda_{\perp}|$. The numerical results of α for different dynamical models and the multiplicative noise model are shown in figure 4. Except extremely near the transition point, the parabolic dependence of α on $\lambda_{\perp}$ holds quite well for all models. The quantitative disagreement in chaotic dynamical systems may be attributed to the contribution of the finite range of temporal correlation which is neglected in the stochastic treatment. Sufficiently near the transition point, α takes a finite value α_0 which is a function of l_0 and l_c. It is

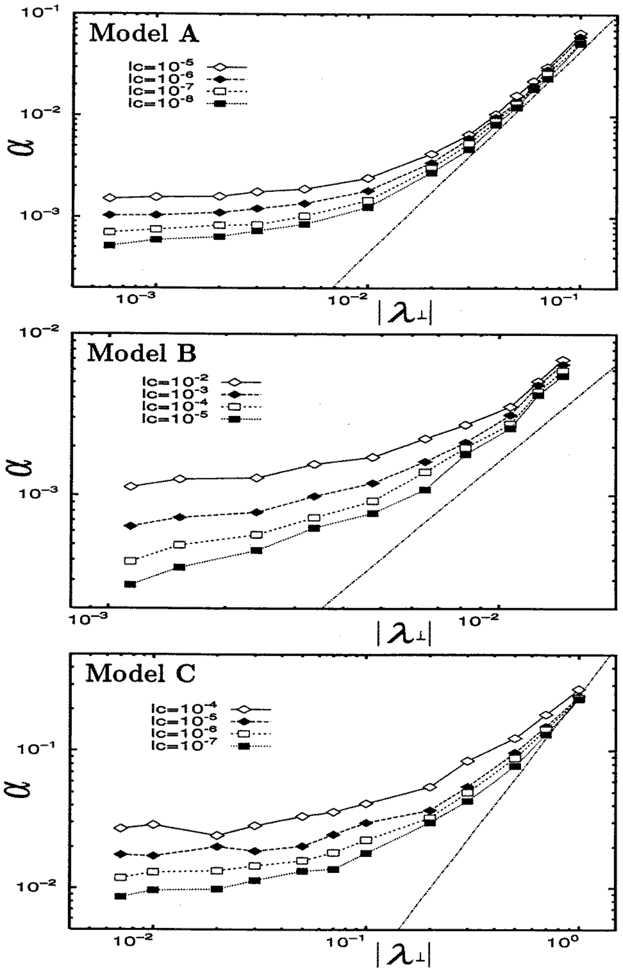

Figure 4. The $|\lambda_\perp|$ dependence of α for the three models for different values of l_c. l_0's are chosen as 0.01, 1 and 1 respectively for models A, B and C. Straight lines are the asymptotic law $\alpha = \lambda_\perp^2/4D_\perp$ with $D_\perp = 0.057$, 0.0154 and 1 respectively for models A, B and C. $D_\perp$'s, except in model C, were calculated with (2.9). Near transition points, the asymptotic law breaks down and α depends on l_c.

easily understood that the deviation from (3.8) is due to the nonlinear fluctuation effect near the transition point. In fact, carrying out numerical simulation for a linear multiplicative noise model, no deviation from (3.8) is observed for any small $|\lambda_\perp|$.

Let us define the characteristic value of $|\lambda_\perp^*|$ via

$$\alpha_0 = \frac{\lambda_\perp^{*\,2}}{4D_\perp}. \tag{3.11}$$

$|\lambda_\perp^*|$ evaluates the characteristic value of $|\lambda_\perp|$. The deviation from the law (3.8) is remarkably observed for $|\lambda_\perp| < |\lambda_\perp^*|$. We observed that α_0 monotonously decreases as l_c is decreased. Numerical results† imply that the relation $|\lambda_\perp^*| = c|\lambda_\perp^0|$ approximately

† In the present work, α_0's are evaluated at the smallest $|\lambda_\perp|$ in each graph in figure 4.

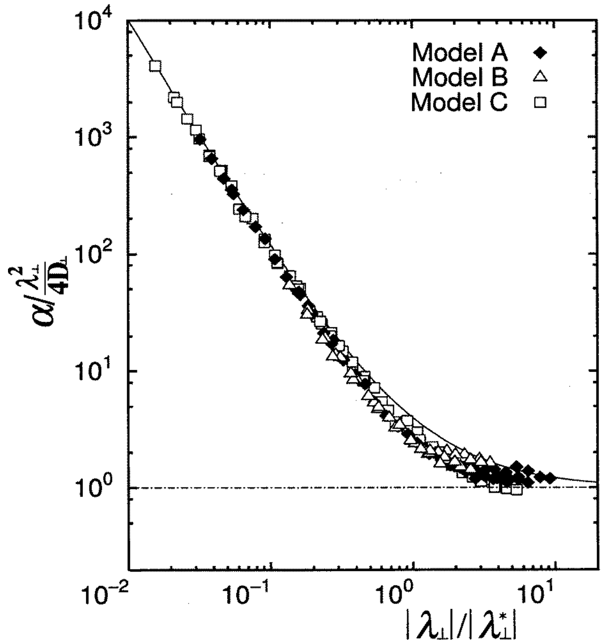

Figure 5. Scaling plots of α versus $|\lambda_\perp|$ for the three models for different values of l_c. All data in figure 4 are drawn. The figure suggests the validity of the scaling law (3.12), and manifests the universality of the function $g(z)$. The full curve is the empirical law $g(z) = (1 + z^{-1})^2$.

holds, where c is a numerical constant between 3 and 4. We thus found two regions of $|\lambda_\perp|$ yielding different asymptotic dependences of α on $\lambda_\perp$, equations (3.8) and (3.11). This may suggest the possibility of the scaling law of α,

$$\alpha = \frac{\lambda_\perp^2}{4D_\perp} g\left(\frac{|\lambda_\perp|}{|\lambda_\perp^*|}\right) \tag{3.12}$$

where $g(z)$ is the scaling function and should have the asymptotic law

$$g(z) = \begin{cases} 1 & (z \gg 1) \\ z^{-2} & (z \ll 1). \end{cases} \tag{3.13}$$

Figure 5 is the scaling plot of α for the three models for different $\lambda_\perp$ and l_c. One finds that the scaling law (3.12) holds quite well for any combination of $\lambda_\perp$ and l_c for the above three different models. In this sense, $g(z)$ is not only a scaling function for a given system but also a universal function valid for the universality class of on–off intermittency transition.

The existence of $\lambda_\perp^*$ is apparently due to the nonlinear fluctuation effect of LTER, $\dot{l}(t)/l(t)$. In order to take into account the nonlinear effect with respect to the deviation l from the invariant manifold, we start with the rigorous equation of motion for $l(t)$ by $\dot{l}(t) = \Lambda_\perp(t)l(t)$. $\Lambda_\perp$ contains nonlinear terms of l. If l is sufficiently small, the statistics of $\Lambda_\perp(t)$ is identical to that of $\Lambda_\perp^0(t)$ in (2.6). Let us divide $\Lambda_\perp(t)$ into two parts,

$$\Lambda_\perp(t) = \lambda_{\perp\mathrm{eff}} + f_{\mathrm{eff}}(t) \tag{3.14}$$

where $\lambda_{\perp\mathrm{eff}}$ is effective TLE under the nonlinear effect and $f_{\mathrm{eff}}(t)$ is the fluctuation part. We assume that $f_{\mathrm{eff}}(t)$ is the Gaussian random force with $\langle f_{\mathrm{eff}}(t)\rangle = 0$ and $\langle f_{\mathrm{eff}}(t) f_{\mathrm{eff}}(0)\rangle = 2D_{\perp\mathrm{eff}}\delta(t)$. This approximation yields $\alpha = \lambda_{\perp\mathrm{eff}}^2/4D_{\perp\mathrm{eff}}$. Since the intensity of the fluctuation of FPT is finite near the transition point, one may replace $D_{\perp\mathrm{eff}}$ by the bare intensity $D_\perp$. Thus we obtain

$$\alpha = \frac{\lambda_{\perp\mathrm{eff}}^2}{4D_\perp}. \tag{3.15}$$

Let τ be the time when l reached the distance l_c starting with the initial distance $l_0(> l_c)$. One obtains

$$\log\frac{l_c}{l_0} = \int_0^\tau \Lambda_\perp(s)\,\mathrm{d}s. \tag{3.16}$$

Since $\{\Lambda_\perp(s)\}$ depends on the initial condition, τ is a function of the initial condition. The coarse-grained transverse Lyapunov exponent in the transient process is evaluated by

$$\bar{\Lambda}_\perp \equiv \frac{1}{\tau}\int_0^\tau \Lambda_\perp(s)\,\mathrm{d}s = \frac{\log(l_c/l_0)}{\tau}. \tag{3.17}$$

So, by defining a characteristic time τ_{eff} in a suitable way, $\lambda_{\perp\mathrm{eff}}$ is given by

$$\lambda_{\perp\mathrm{eff}} = \frac{\log(l_c/l_0)}{\tau_{\mathrm{eff}}}. \tag{3.18}$$

If τ_{eff} is replaced by $\langle\tau\rangle_0$, one obtains $\lambda_{\perp\mathrm{eff}} = \lambda_\perp$, which implies no contribution from the nonlinear fluctuation. The deviation of τ_{eff} from $\langle\tau\rangle_0$ especially near the transition point changes the law (3.8).

4. Concluding remarks

In this paper we have discussed the statistics of the transient below the on–off intermittency transition. The first passage time sensitively depends on the initial condition, which is the origin of the distribution of first passage time. We empirically found a scaling law for the characteristic first passage time (3.12). The scaling law reflects the importance of the nonlinear fluctuation of the intermittency variable in the transient process. Phenomenologically introducing a characteristic time τ_{eff}, we briefly discussed the renormalization of the effective transverse Lyapunov exponent by nonlinear fluctuation. At the present stage, we have no theory to determine τ_{eff}. The determination of τ_{eff} may clarify the validity of the scaling law. Work in determining τ_{eff} is therefore desirable.

The on–off intermittency is a quite universal phenomenon observed when a particular chaotic state looses its stability. Many studies have been devoted mainly to the statistics of the on–off intermittency. On the other hand, the interrelation between the onset of on–off intermittency and the geometrical structure in the state space has recently been extensively discussed by Ott *et al* [7, 21].

Acknowledgments

The authors thank Dr H Hata and Dr S Miyazaki for valuable discussions, especially on the statistics of the characteristic first passage time near the on–off intermittency transition. This work was partially supported by Grant-in-Aid for General Scientific Research (no 40156849) from the Ministry of Education and Culture, Japan.

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